Stackelberg duopoly with symmetric firms and linear demand

Consider a market with two firms, Firm 1 and Firm 2, that produce a homogeneous good. The market demand function is given by $P(Q) = a - bQ$, where $Q = q_1 + q_2$ is the total quantity supplied to the market, and $a, b > 0$. Both firms have the same constant marginal cost of production, c, where $a > c$.

In this Stackelberg duopoly model, Firm 1 is the leader and Firm 2 is the follower. Firm 1 chooses its output q_1 in the first period, and Firm 2 observes this choice and then decides its output q_2 in the second period.

1. Derive Firm 2's reaction function $q_2 = R_2(q_1)$.
2. Using Firm 2's reaction function, determine Firm 1's profit-maximizing output q_1^*.
3. Calculate the equilibrium quantities q_1^* and q_2^*.
4. Determine the equilibrium market price p^*.
5. Compute the equilibrium profits for both firms, Π_1^* and Π_2^*.
6. Compare the Stackelberg equilibrium quantities, price, and profits with those in a Cournot duopoly where both firms choose their quantities simultaneously.

Solution

1. We start with the second firm:

$$\max_{q_2} \Pi_2 = (P(q_1+q_2)-c)\,q_2 = (a-b(q_1+q_2)-c)\,q_2$$

$$\text{FOC:} \quad \frac{\partial \Pi_2}{\partial q_2} = 0 \iff a - 2bq_2 - bq_1 - c = 0$$

$$\iff q_2^* = R_2(q_1) = \frac{a-bq_1-c}{2b}$$

2. In the 1st period (firm 1 chooses q_1 knowing that firm 2 will react to it in the 2nd period according to its reaction function $q_2 = R_2(q_1)$):

$$\max_{q_1} \Pi_1 = (P(q_1+q_2)-c)\,q_1 = (a-b(q_1+R_2(q_1))-c)\,q_1$$

$$\max_{q_1} \Pi_1 = aq_1 - bq_1^2 - bR_2(q_1)q_1 - cq_1$$

$$\text{FOC:} \quad \frac{\partial \Pi_1}{\partial q_1} = 0 \iff a - 2bq_1 - bR_2(q_1) - bq_1R_2'(q_1) - c = 0$$

$$\iff a - 2bq_1 - b\left[\frac{a-bq_1-c}{2b}\right] + bq_1 \cdot \frac{1}{2} - c = 0$$

$$\iff a - 2bq_1 - \frac{a}{2} + \frac{bq_1}{2} + \frac{c}{2} + bq_1 \cdot \frac{1}{2} - c = 0$$

$$\iff \frac{a}{2} - \frac{c}{2} - bq_1 = 0$$

$$\iff \left(\frac{a-c}{2}\right) - bq_1 = 0 \iff q_1^* = \frac{a-c}{2b}$$

3. **Given q_1^* we solve for q_2**

$$q_2^* = \frac{a-c}{2b} - \frac{1}{2}q_1^* = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{2b}\right) = \left(\frac{a-c}{4b}\right)$$

Therefore $q_1^* > q_2^*$

4.

$$q_1^* + q_2^* = \frac{a-c}{2b} + \frac{a-c}{4b} = \frac{3(a-c)}{4b}$$

$$p^* = a - bQ^* = a - b\left[\frac{3(a-c)}{4b}\right]$$

5. **The equilibrium profits of both firms:**

$$\Pi_1^* = (p^* - c)\, q_1^* = \left(\frac{a+3c}{4} - c\right)\left(\frac{a-c}{2b}\right) = \left(\frac{a-c}{4}\right)\left(\frac{a-c}{2b}\right) = \left(\frac{a-c}{2b}\right)^2 = \frac{(a-c)^2}{9b}$$

$$\Pi_2^* = (p^* - c)\, q_2^* = \left(\frac{a+3c}{4} - c\right)\left(\frac{a-c}{4b}\right) = \left(\frac{a-c}{4}\right)\left(\frac{a-c}{4b}\right) = \left(\frac{a-c}{4b}\right)^2 = \frac{(a-c)^2}{16b}$$

6. Comparison with Cournot

   (a) $q_1^* > q_2^*$ (the leader produces more)

   (b) $p^* > c$ (There will be a deadweight loss)

   (c) $\Pi_1^* > \Pi_2^*$ (the leader has higher profits, there is an advantage of being the first to choose)

   (d) $Q^* > Q^{cournot} \Rightarrow p^* < p^{cournot}$

   The leader knows that by increasing q_1 the follower will reduce q_2 (quantities are strategic substitutes). Stackelberg leads to a more competitive equilibrium than Cournot